Illustrative Mathematics

4.NF Money in the piggy bank

Alignment 1: 4.NF.A

Alicia opened her piggy bank and counted the coins inside. Here is what she found:

- 22 pennies
- 5 nickels
- 5 dimes
- 8 quarters
- a. How many coins are in Alicia's piggy bank?
- b. What fraction of the coins in the piggy bank are dimes?
- c. What is the total value of the coins in the piggy bank? Give your answer in cents: for example \$2.35 is 235 cents.
- d. What fraction of the total value of the coins in the piggy bank is made up of dimes?

Commentary:

This task is designed to help students focus on the whole that a fraction refers to. It provides a context where there are two natural ways to view the coins:

- · As equal parts of the set of coins in the piggy bank, and
- As money so each coin is assigned its monetary value.

These two points of view are very different: according to the first point of view, all coins are considered equal parts of the collection of coins, with a penny being one part of the collection just as a nickel or dime is one part. From the second point of view, a penny is one cent while a nickel is five cents and in particular a nickel is viewed as equivalent to five pennies. The important thing to realize here is that two different fractions can describe the same situation depending on what you choose to be the whole.

The instructor may wish to ask the students to compare the fractions which they find in parts (b) and (d): which is larger? This falls outside of the 4-NF.2 standard which says "comparisons are valid only when the two fractions refer to the same whole." In this case, for the fraction from part (b) the whole is the collection of coins while for the fraction in part (d) the whole is the total value of coins in the piggy bank. However, once the students have found the fractions $\frac{1}{8}$ (or $\frac{5}{40}$) and $\frac{50}{297}$ in parts (b) and (d) it is a legitimate question to ask which is a larger number. In other words, it is not possible to compare $\frac{1}{8}$ of the coins with $\frac{50}{297}$ of the money but it is possible to compare the numbers $\frac{1}{8}$ and $\frac{50}{297}$. One elegant way to do this is the following: note that

$$\frac{5}{40} = \frac{50}{400}$$

and since $\frac{1}{400}$ is smaller than $\frac{1}{297}$ (cutting up a whole into 400 pieces results in smaller pieces than cutting up the same whole into 297 pieces) we can see that

$$\frac{5}{40} < \frac{50}{297}$$
.

We can even make sense of this comparison in the context: the value of the dimes, as a fraction of the value of all the coins in the piggy bank, is greater than the number of dimes, as a fraction of all the coins in the piggy bank.

This task could be used for assessment but it is mainly designed to provide a rich context for students to work on their understanding of fractions

Solution: 1

a. To find the total number of coins in Alicia's piggy bank, we need to add the number of pennies, nickels, dimes, and quarters:

$$22 + 5 + 5 + 8$$
.

A good way to do this would be to group the individual terms in a convenient way:

$$22 + 5 + 5 + 8 = 22 + (5 + 5) + 8$$
$$= (20 + 2) + 10 + 8$$
$$= 20 + 10 + (2 + 8)$$
$$= 30 + 10$$
$$= 40.$$

So there are 40 total coins in the piggy bank.

b. We know from part (a) that there are 40 coins in the piggy bank. We also know that 5 of these coins are dimes. So this means that the fraction of coins in the piggy bank which are dimes is

$$\frac{5}{40}$$

It is appropriate to record the fraction of dimes as $\frac{5}{40}$ without "reducing" it, but students can also write it as $\frac{1}{8}$. Here the only advantage would be that it is a little easier to wrap one's mind around $\frac{1}{8}$, but in part (e) of the task, there is a strong advantage to seeing the equivalence between $\frac{5}{40}$ and $\frac{1}{8}$.

c. To find the total value of the coins in piggy bank we can first find the value of the pennies, nickels, dimes, and quarters which is shown in the table below:

denomination	number of coins	value of coins in cents
penny	22	22
nickel	5	$5 \times 5 = 25$
dime	5	$5 \times 10 = 50$
quarter	8	$8 \times 25 = 200$

Now we add the values for each denomination to find to get the total amount of money in the piggy bank;

$$22 + 25 + 50 + 200 = 297$$
.

So the total value of the coins in the piggy bank is 297 cents or 2 dollars and 97 cents.

d. The total value of the dimes, from the table in part (c), is 50 cents. The total amount of money in the piggy bank, also from part (c), is 297 cents. So the fraction of the money in the piggy bank which comes from the dimes is $\frac{50}{297}$.